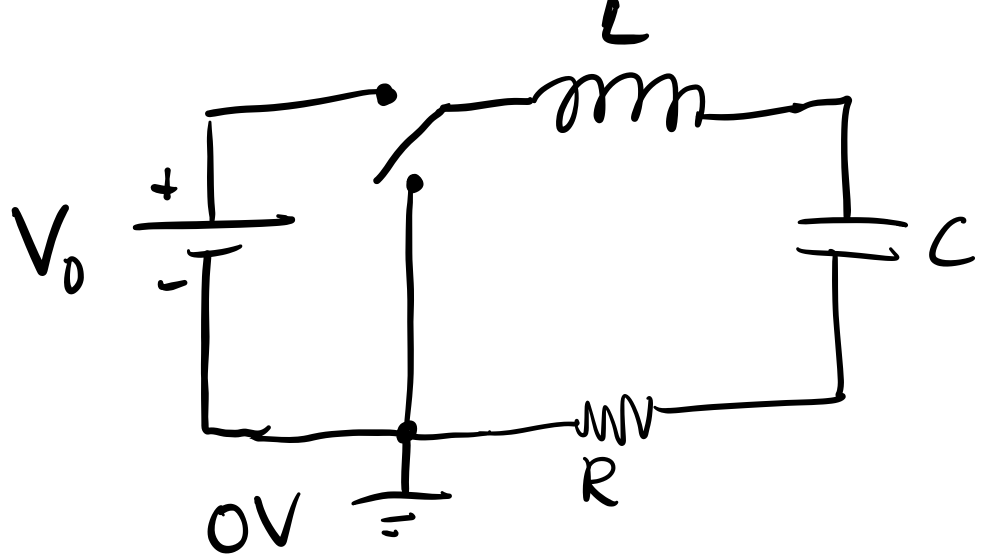
PHYS 231 - Sept. 25, 2023

Assign #1 is online.
Due Wed. Oct. 4.

No classes $\underbrace{\text{Oct. 2}}_{\text{T\&R}}$ & $\underbrace{\text{Oct. 9}}_{\text{Thanksgiving.}}$

Today: Series LRC circuit (Lab #4)



When the switch is in the "up" position, K.V.R. requires:

$$V_0 - V_L - V_C - V_R = 0$$

$$V_0 - L\frac{d^2q}{dt^2} - \frac{q}{C} - R\frac{dq}{dt} = 0$$

$$\therefore \boxed{\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{V_0}{L}} \quad ⓐ$$

Second order diff. eq'n. ⇒ Solve for $q(t)$.

You will encounter & solve this diff. eq'n in MATH 225 & PHYS 216 next term.

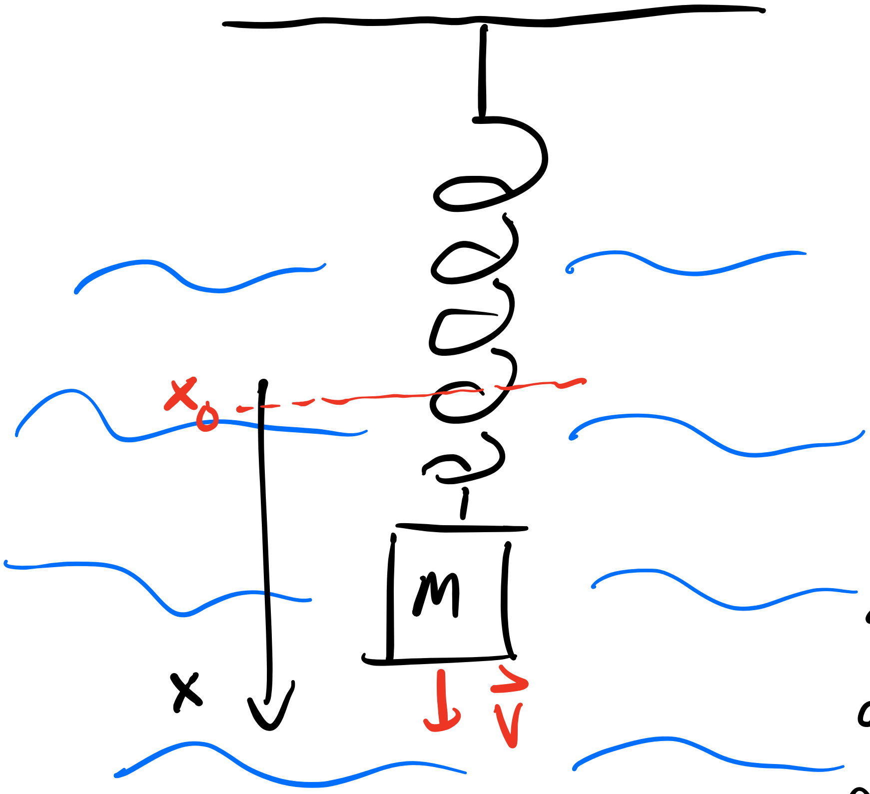
If you like, see full sol'n on PHYS 231 website (optional reading, not required).

Start by assuming $q(t) = q_0 + q_n(t)$

...

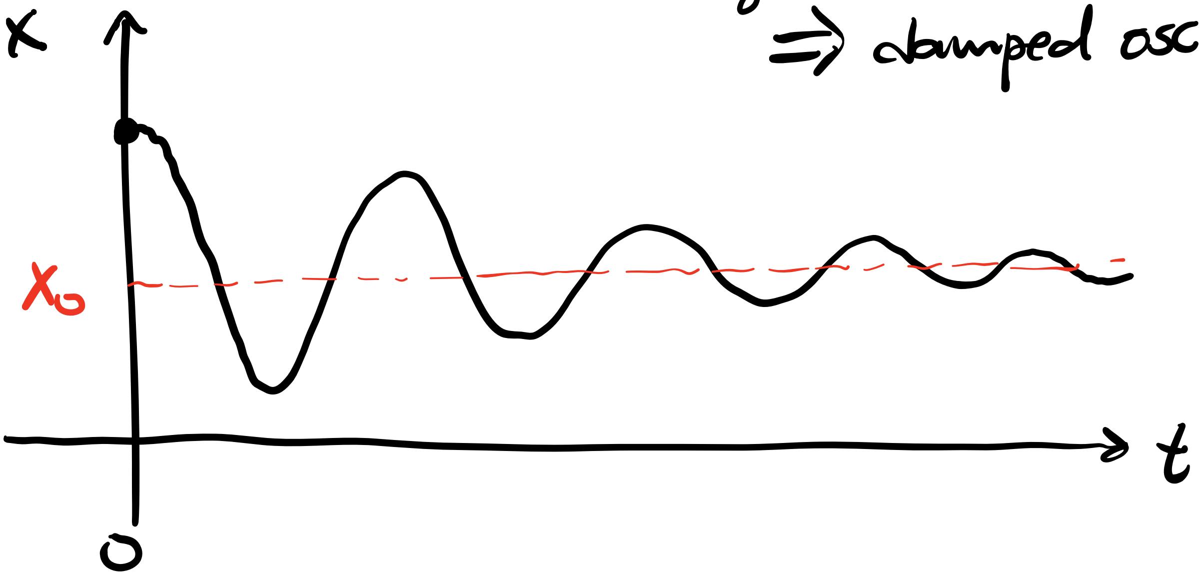
Consider the mechanical analogy to the LRC circuit:

Mass hanging from a ceiling by a spring suspended in a viscous fluid.

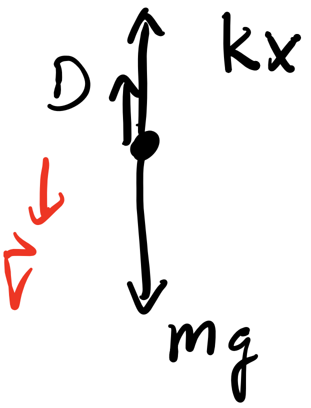


If we stretch spring past its equil. & then release the mass from rest, the mass will osc. & the amp. of osc. will decrease w/ time due to viscous drag.

⇒ damped osc.



FBD for mass on spring



assume that the drag force is given by $D = bv$

drag coefficient

From Newton's 2nd Law

$$\underbrace{ma}_{\text{net force}} = mg - bv - kx$$

$$v = \frac{dx}{dt} \qquad a = \frac{d^2x}{dt^2}$$

$$\therefore mg = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx$$

divide by m:

$$\boxed{g = \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x}$$ ⓑ

Mathematically Eq'ns ⓐ & ⓑ are identical. ∴ sol'n for $x(t)$ in ⓑ gives us insight into the sol'n for $q(t)$ in ⓐ.

For the mass on a spring, can find the equil. position that we reach when $t \to \infty$ by setting $\frac{dx}{dt} = \frac{d^2x}{dt^2} = 0$ in ⓑ.

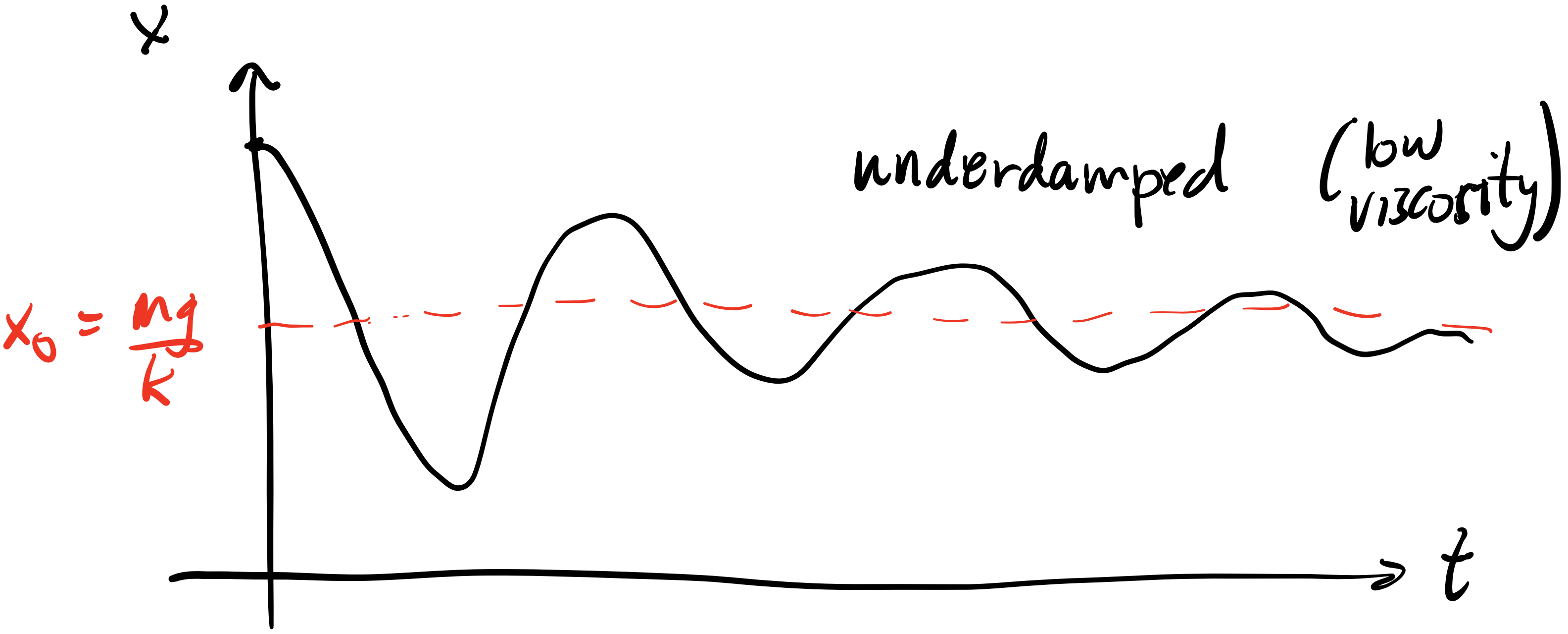
equil. position

Get $g = \frac{k}{m} x_0 \Rightarrow x_0 = \frac{mg}{k}$

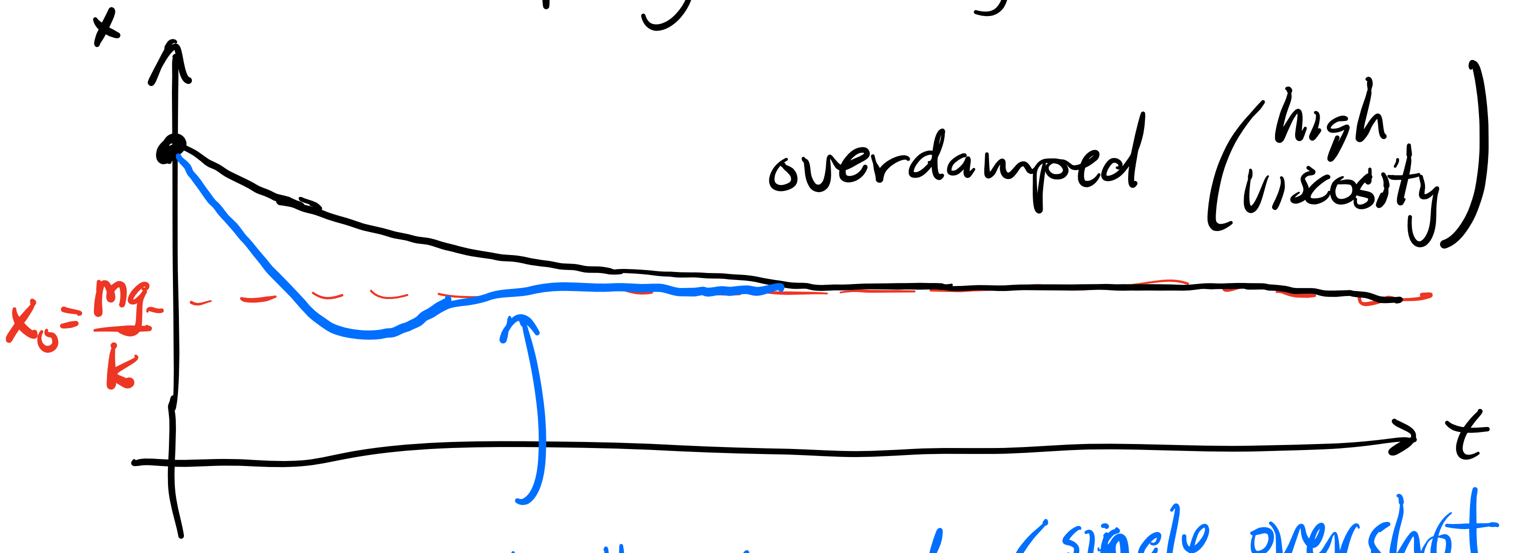
For the LRC circuit, can find equil. charge in the same way ($\frac{dq}{dt} = \frac{d^2q}{dt^2} = 0$ in Eq'n ⓐ).

$$\Rightarrow \frac{1}{\cancel{L}C} q_0 = \frac{V_0}{\cancel{L}} \qquad \boxed{q_0 = C V_0}$$

Mass on a spring when oil



Mass on a spring in honey



critically-damped (single overshoot & fastest approach to equil.)

For the LRC circuit, we will focus on the underdamped case.

For underdamping, we require:

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}} \rightsquigarrow$$ i.e. R small for underdamping.